

7.1

Angle Relationships in Triangles

polygon

- closed figure made up of line segments

The triangle is the simplest type of **polygon**. The structures of buildings and machines often contain triangles. Triangular shapes can be particularly strong and rigid since the shape of a triangle cannot change unless the length of a side changes. In this section, you will learn about some of the other properties of triangles.



Tools

- ruler
- protractor

vertex

- point where two or more sides meet

interior angle

- angle formed on the inside of a polygon by two sides meeting at a vertex

exterior angle

- angle formed on the outside of a geometric shape by extending one of the sides past a vertex

Literacy Connections

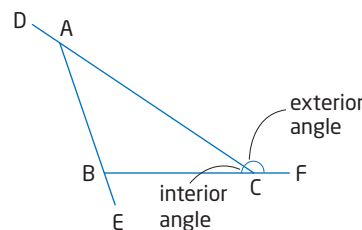
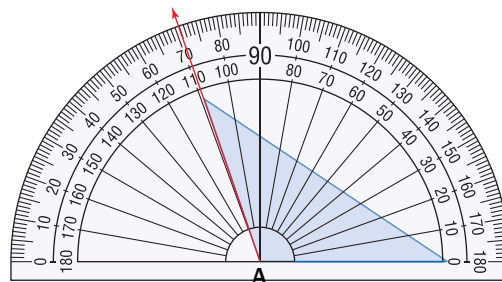
The plural of *vertex* is *vertices*. This plural form comes from Latin.

Investigate

How are the interior and exterior angles of a triangle related?

Method 1: Use Pencil and Paper

1. Draw a large triangle. Label the first **vertex** A, the second one B, and the third one C.
2. Measure each of the **interior angles** of your triangle with a protractor. Mark these measurements on your diagram.
3. At each vertex, extend one side of the triangle to form an **exterior angle**. Measure each of these angles, and mark the measurements on your diagram.



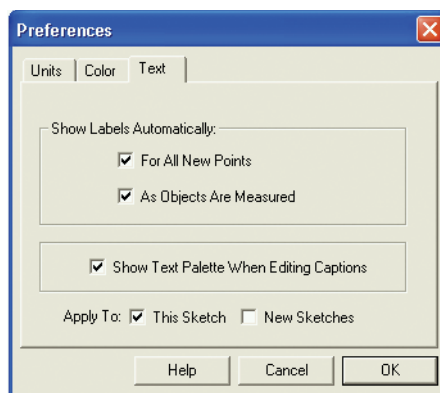
4. Find the sum of the exterior angles. Compare the sum you found to those calculated by your classmates.
5. **Reflect** Do you think the sum of the exterior angles is the same for every triangle? Explain your reasoning.
6. How are the exterior angle and the interior angle at a vertex related?

7. Reflect

- Describe the relationship between the exterior angle at a vertex of a triangle and the interior angles at the other two vertices.
- Will this relationship apply to the exterior angles of all triangles? Explain.

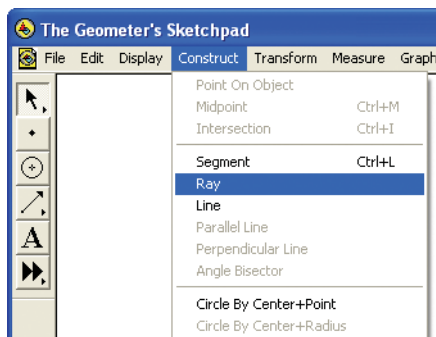
Method 2: Use *The Geometer's Sketchpad*®

- Turn on automatic labelling of points. From the **Edit** menu, choose **Preferences**. Click on the **Text** tab, check **For All New Points**, and click on **OK**.



- Use the **Point Tool** to create three points on the screen.

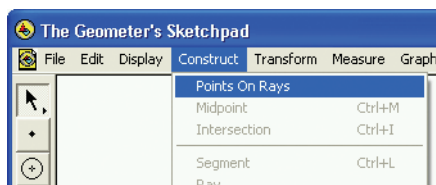
- Select point A and point B. From the **Construct** menu, choose **Ray**. Use the same method to construct **rays** from B to C and from C to A.



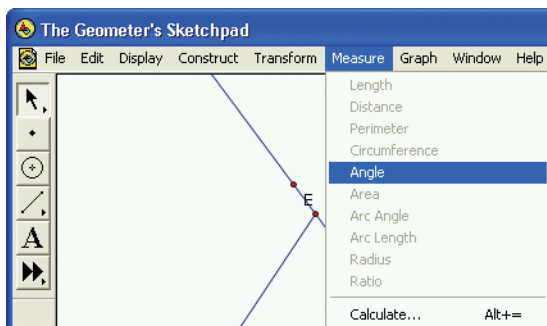
ray

- a part of a line with one endpoint

- Select the three rays. From the **Construct** menu, choose **Points on Rays**. If necessary, drag each point to a location outside the triangle.



- To measure $\angle FAB$, select points F, A, and B in that order. From the **Measure** menu, choose **Angle**. Measure $\angle DBC$ and $\angle ECA$ the same way.



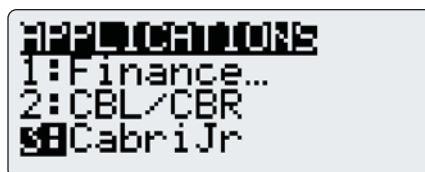
6. From the **Measure** menu, choose **Calculate**. A calculator window will appear. Click on the measure for $\angle BAF$. The measure will appear in the calculator window. Click $+$ on the calculator; then, click on the measure for $\angle DBC$. Click $+$ again, and add the measure of $\angle ECA$.



7. How are the exterior angle and the interior angle at a vertex related?
8. Make a hypothesis about the sum of the exterior angles in any triangle.
9. Drag one of the vertices around the screen. What happens to the sum of the exterior angles? Try moving the other two vertices as well.
10. **Reflect** What can you conclude about the sum of the exterior angles of any triangle? Do your observations support your hypothesis? Explain.
11. Make a hypothesis about the relationship between the exterior angle at a vertex of a triangle and the interior angles at the other two vertices.
12. Use the Measure and Calculate tools to test your hypothesis.
13. **Reflect** Is your hypothesis correct? Explain.

Method 3: Use a Graphing Calculator

1. Press APPS and select **CabriJr**. Press ENTER when the title screen appears.



2. If you need to clear the screen, press Y= to display the **F1** menu, and select **New**.



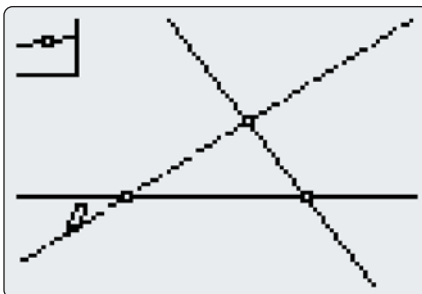
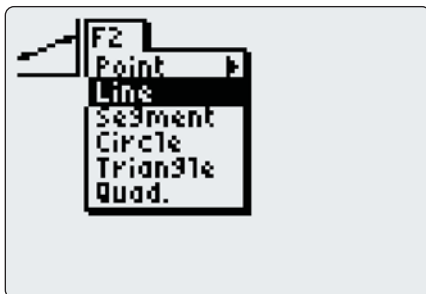
■ TI-83 Plus or TI-84
graphing calculator

Technology Tip

The position of **CabriJr** on the **APPS** screen depends on what other applications have been installed.

You can download Cabri® Jr. by following the links at www.mcgrawhill.ca/links/principles9.

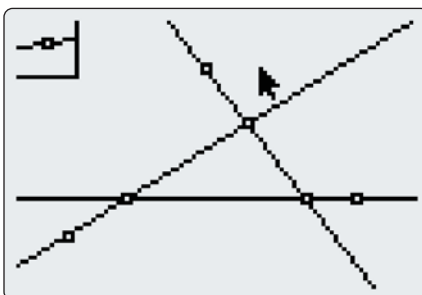
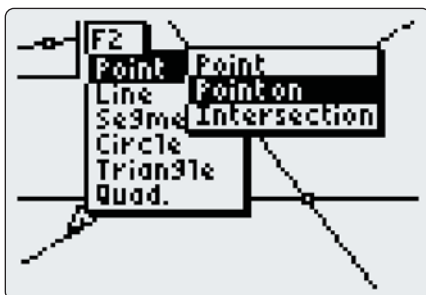
3. To draw a triangle, press **WINDOW** to display the **F2** menu, and select **Line**. Move the pencil cursor to where you want the first **vertex** and press **ENTER**. Move the cursor to the location for the second vertex and press **ENTER** twice. Position the third vertex in the same way. Then, move the cursor back to the first vertex and press **ENTER** again.



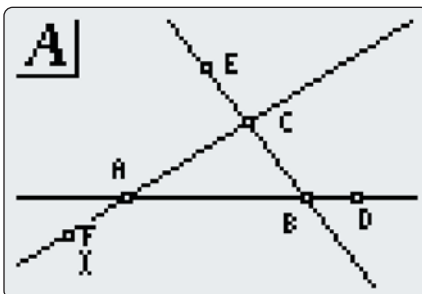
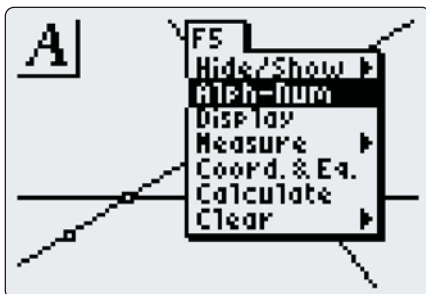
Technology Tip

In Cabri® Jr., you can use the keys directly under the screen as function keys without pressing **ALPHA** first.

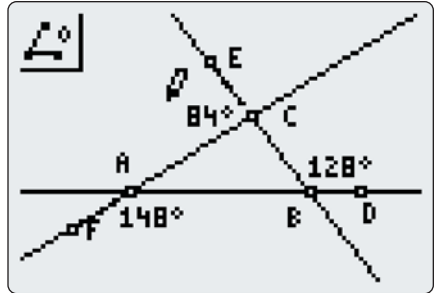
4. Press **WINDOW** to display the **F2** menu. With the cursor on **Point**, press **RIGHT**. Select **Point on** from the submenu. To place a point for an **exterior angle**, move the cursor onto the portion of a line outside the triangle and press **ENTER**. Place similar points on the other two lines.



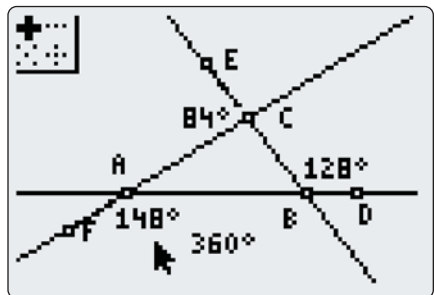
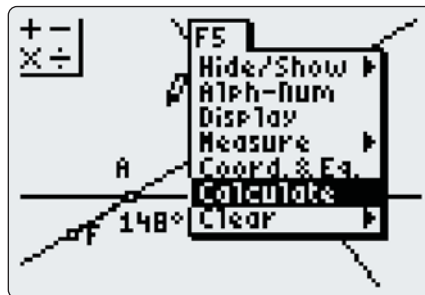
5. Press **GRAPH** to display the **F5** menu, and select **Alph-Num**. To label a point, move the cursor near the point and press **ENTER** **ALPHA**. Press the key for the letter you want; then, press **ENTER** to lock the label in place. To move the label, press **CLEAR**, and move the cursor close to the letter until it is underlined. Press **ALPHA**, use the cursor keys to drag the label to a new location, and press **ENTER**.



6. To measure angles, press **GRAPH** to display the **F5** menu, highlight **Measure**, and press **▶**. Select **Angle** from the submenu. Select the three points that define the angle in order (the vertex of the angle is the second point). To select each point, move the cursor to it and press **ENTER**. Use the cursor keys to drag the measurement to a convenient location and press **ENTER**. Measure the three exterior angles of the triangle.



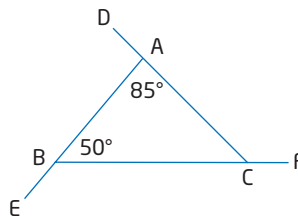
7. Find the sum of the three exterior angles. Press **GRAPH** to display the **F5** menu and select **Calculate**. Select each measurement by moving the cursor to it and pressing **ENTER** when the measurement is underlined. Then, press **+** and drag the total to an empty part of the screen.



8. Make a hypothesis about the sum of the exterior angles of any triangle.
9. Press **CLEAR** and move the cursor to one of the vertices. When the vertex starts flashing, press **ALPHA**. Now, use the cursor keys to drag the vertex to various new locations. Watch the sum of the exterior angles as you move the vertex. Try moving the other two vertices as well.
10. **Reflect** What can you conclude about the sum of the exterior angles of any triangle? Do your observations support your hypothesis? Explain.
11. How are the exterior angle and the interior angle at a vertex related?
12. Make a hypothesis about the relationship between the exterior angle at a vertex of a triangle and the interior angles at the other two vertices.
13. Use Cabri® Jr. to test your hypothesis.
14. **Reflect** Is your hypothesis correct? Explain.

Example 1 Measures of the Exterior Angles of a Triangle

Find the measures of the exterior angles of $\triangle ABC$.



Solution

At vertex A and at vertex B, the interior and exterior angles together form a 180° angle.

$$\angle DAB + \angle CAB = 180^\circ$$

$$\angle DAB = 180^\circ - \angle CAB$$

$$\angle DAB = 180^\circ - 85^\circ$$

$$\angle DAB = 95^\circ$$

$$\angle EBC + \angle ABC = 180^\circ$$

$$\angle EBC = 180^\circ - \angle ABC$$

$$\angle EBC = 180^\circ - 50^\circ$$

$$\angle EBC = 130^\circ$$

There are two ways to use the properties of exterior angles to find the measure of $\angle ACF$.

Method 1

Since the exterior angle at a vertex of a triangle is equal to the sum of the interior angles at the other two vertices,

$$\angle ACF = \angle CAB + \angle ABC$$

$$= 85^\circ + 50^\circ$$

$$= 135^\circ$$

Method 2

Since the sum of the exterior angles of a triangle is 360° ,

$$\angle ACF + \angle DAB + \angle EBC = 360^\circ$$

$$\angle ACF = 360^\circ - \angle DAB - \angle EBC$$

$$\angle ACF = 360^\circ - 95^\circ - 130^\circ$$

$$\angle ACF = 135^\circ$$

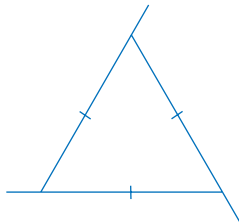
The measures of the three exterior angles are $\angle DAB = 95^\circ$, $\angle EBC = 130^\circ$, and $\angle ACF = 135^\circ$.

Literacy Connections

In angle names with three letters, the middle letter is always the vertex of the angle.

Example 2 Exterior Angles of an Equilateral Triangle

What is the measure of each exterior angle of an equilateral triangle?



Solution

Method 1: Calculate the Measure of the Interior Angles First

An equilateral triangle is also **equiangular**. The sum of the interior angles of any triangle is 180° . Since the three interior angles are equal, each one must measure $\frac{180^\circ}{3}$ or 60° .

At each vertex, the interior angle and the exterior angle together make a 180° angle.

Therefore, the measure of each exterior angle is $180^\circ - 60^\circ$ or 120° .

You get the same result by using the property that each exterior angle is equal to the sum of the interior angles at the other two vertices.

Method 2: Apply the Properties of Exterior Angles

At each vertex, the interior angle and the exterior angle together make a 180° angle. Since the interior angles are equal, the three exterior angles must also be equal.

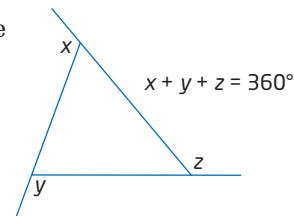
The sum of the exterior angles is 360° . Therefore, each exterior angle of an equilateral triangle must measure $\frac{360^\circ}{3}$ or 120° .

equiangular

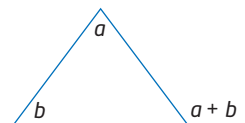
- having all angles equal

Key Concepts

- The sum of the exterior angles of a triangle is 360° .

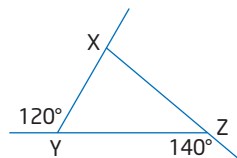


- The exterior angle at each vertex of a triangle is equal to the sum of the interior angles at the other two vertices.

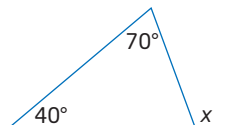


Communicate Your Understanding

- C1** Find the measure of the exterior angle at vertex X. Explain your reasoning.



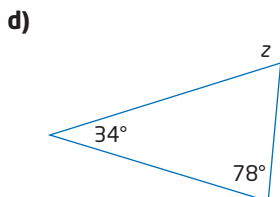
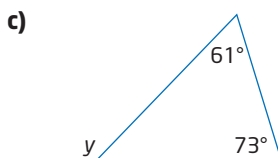
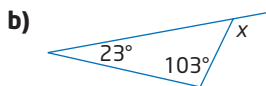
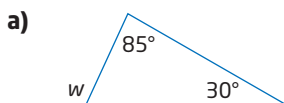
- C2** Helena says this exterior angle measures 110° . Is she right? Explain.



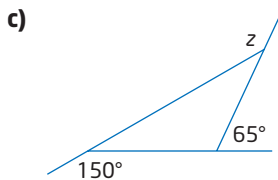
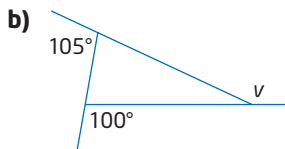
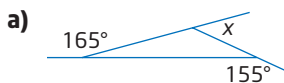
Practise

For help with questions 1 to 3, see Example 1.

1. Find the measure of each exterior angle.



2. Find the measure of each unknown exterior angle.

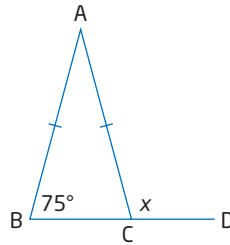


3. If the measures of two of the exterior angles of a triangle are 70° and 120° , the measure of the third exterior angle is
- A** 10°
 - B** 70°
 - C** 170°
 - D** 190°

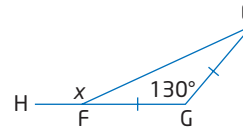
Connect and Apply

4. Find the measure of the exterior angle labelled x for each isosceles triangle.

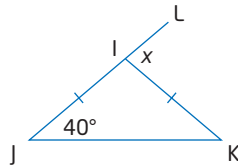
a)



b)

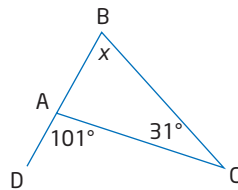


c)

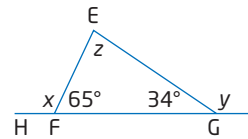


5. Find the measure of each unknown angle.

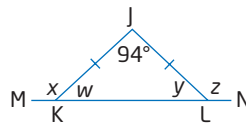
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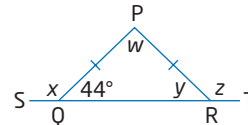
b)



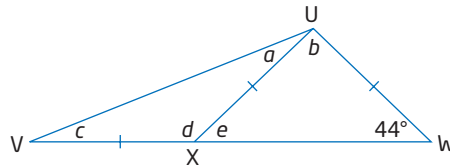
c)



d)



e)



6. One exterior angle of an isosceles triangle measures 140° . Find the possible measures for the other two exterior angles.

7. Calculate the mean measure of the exterior angles of a triangle.

8. What types of triangles have some exterior angles equal?

9. a) Explain why a triangle cannot have two obtuse interior angles.

- b) Can a triangle have three obtuse exterior angles? Justify your answer.

